Localization and Metal-Insulator Transition in Multi-Layer Quantum Hall Structures

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Abstract

We study the phase structure and Hall conductance quantization in weakly coupled multi-layer electron systems in the integer quantum Hall regime. We derive an effective field theory and perform a two-loop renormalization group calculation. It is shown that (i) finite interlayer tunnelings (however small) give rise to successive metallic and insulating phases and metal-insulator transitions in the unitary universality class. (ii) The Hall conductivity is not renormalized in the metallic phases in the 3D regime. (iii) The Hall conductances are quantized in the insulating phases. In the bulk quantum Hall phases, the effective field theory describes the transport on the surface.

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The quantum Hall effect (QHE) in a two-dimensional electron gas (2DEG) has led to many new physical concepts and principles [1]. The main part of the phenomenology is that in strong magnetic fields, a 2DEG exhibits continuous zero-temperature phase transitions between successive quantum Hall states of vanishing dissipation and quantized Hall resistances. It is interesting to ask what happens to the physics associated with the QHE in dimensions greater than two [2]. Experimentally, two classes of quasi-three-dimensional materials have been found to show integer-quantized Hall plateaus: multi-layer quantum wells or superlattices formed by GaAs/AlGaAs graded heterostructures [3,4] and molecular crystals $(TMTSF)_2X$ ($X=PF_6$,ClO_4) [5]. In this paper, we concentrate on the former which is a natural generalization of the QHE above two dimensions. Thus, the changes in the phase structure and the properties of the phase transitions in quantum Hall layers coupled by weak interlayer tunnelings are the concerns of the present paper. Moreover, we focus on the integer quantum Hall regime and ignore the effects of electron-electron interactions [6].

We shall follow the approach of Chalker and Dohmen who generalized the Chalker-Coddington network model [7] for the integer QHE (IQHE) in a 2DEG to layers of networks coupled by interlayer tunnelings [8]. The advantage of this approach is that the single layer network model is known to correctly describe the universality class of the 2D integer quantum Hall transitions. Chalker and Dohmen performed numerical transfer matrix calculations and demonstrated the existence of three phases: insulator, metal, and quantized Hall conductor, and extended surface states in the quantized Hall state. In this paper we provide an analytical treatment using the effective field theory representation of the network model [9–11]. We first show that the long wavelength transport properties are governed by a 3D anisotropic unitary nonlinear σ-model (NLσM). The anisotropic couplings are the dissipative conductivities, whereas the Hall conductivity enters as a coupling to the layered sum of the 2D topological term. The renormalization group (RG) is then used to study the crossover between two and three dimensions. In the 3D regime, the RG flow equations for the conductances are calculated to two-loop order to determine the phase structure in the plane spanned by the Fermi energy and the interlayer tunneling. The results show that a

finite interlayer coupling (however weak) leads to metallic and insulating phases and metal-insulator transitions in the unitary universality class. Furthermore, the Hall conductivity is found to be unrenormalized by localization effects in the 3D regime. We show that the Hall conductance is quantized in the insulating phases provided that the above results hold to all orders in the RG. Finally, we demonstrate that in the quantum Hall phases, the field theory reduces to the one appropriate for the coupled edge states on the surface.

Following Ref. [10], the Hamiltonian for N layered networks in the (x,y)-plane coupled in the z-direction by interlayer tunneling $t_\perp$ is,

$$\begin{aligned} H_0 = & \sum_{x,z}(-1)^x \int dy \psi^\dagger(x,y,z)\left[i\partial_y - V_{x,y,z}\right]\psi(x,y,z) \\ & -\sum_{x,z}\int dy\ t_x \left[\psi^\dagger(x+1,y,z)\psi(x,y,z) + \text{h.c.}\right] \\ & -\sum_{x,z}\int dy\ t_\perp \left[\psi^\dagger(x,y,z+1)\psi(x,y,z) + \text{h.c.}\right]. \end{aligned} \tag{1}$$

Here $\psi^\dagger$ creates an electron traversing the edges of the Hall droplets as modeled by the links of the network. $t_x = t[1-\delta(-1)^x]$, where δ measures the distance of the Fermi energy (E_F) relative to the center of the Landau level (E_c), represents the quantum tunneling amplitudes at the saddle points of the random potential, *i.e.* at the nodes of the network. V is a local random variable that generates the link Aharonov-Bohm phases, $< V_{x,y,z}V_{x',y',z'} >= U\delta_{x,x'}\delta_{z,z'}\delta(y-y')$.

For $t_\perp = 0$, Eq. (1) describes N decoupled 2D networks, each undergoes a quantum Hall transition as δ is varied. In this case, quench averaging over V and regarding y as the Euclidean time τ, it has been shown that the original network model corresponds to a half-filled 1D U(2n) Hubbard model in the limit $n \to 0$ [10]. The 2D quantum Hall transition is then equivalent to the dimerization transition of the Hubbard chain [10]. Generalizing to $t_\perp \neq 0$, we obtain the generating functional $Z = \int\mathcal{D}[\bar\psi,\psi]\exp[\int d\tau \sum_{xz}(i\eta S_p\bar\psi\psi - H_0(\bar\psi,\psi))]$ in the form of a 2+1-dimensional Euclidean action if we let $\psi_p \to \psi_p(i\psi_p)$ and $\bar\psi_p \to -i\bar\psi_p(\bar\psi_p)$ for even (odd) x [10],

$$S = \int d\tau \left[\sum_{x,z,a}\bar\psi_a\partial_\tau\psi_a + H|_{\psi^\dagger(\psi)\to\bar\psi(\psi)}\right]. \tag{2}$$

Here η is a positive infinitesimal, $a = (\alpha, p)$ are the replica index $\alpha = 1, \ldots, n$ and energy index $p = +(-)$ for the advanced (retarded) channels, and $S_p \equiv \mathrm{sgn(p)}$. The resulting Hamiltonian H in Eq. (2) corresponds to an interacting quantum theory in two spatial dimensions,

$$\begin{aligned} H = & -\sum_{x,z} t_x \left[\psi_a^\dagger(x+1,z)\psi_a(x,z) + \text{h.c.}\right] + \frac{U}{2}\sum_{x,z}\left[\psi_a^\dagger(x,z)\psi_a(x,z)\right]^2 \\ & + \sum_{x,z} it_\perp(-1)^x \left[\psi_a^\dagger(x,z+1)\psi_a(x,z) + \text{h.c.}\right] - \eta\sum_{x,z}(-1)^x S_p \psi_a^\dagger\psi_a, \end{aligned} \tag{3}$$

where sums over repeated indices are implied. Note that Eq. (3) is not the usual quasi-1D U(2n) Hubbard model for the form of the interchain couplings.

We now derive the effective low energy, long wavelength theory. The partition function for Eq. (3) can be written as $Z = \int \mathcal{D}[\bar\psi,\psi]\mathcal{D}[\Delta]\exp(-S)$,

$$\begin{aligned} S &= \int d\tau \sum_{x,z} -\eta(-1)^x S_p \bar\psi_a\psi_a + S_0 + S_I + S_\perp \\ S_0 &= \int d\tau \sum_{x,z}\left[\bar\psi_a\partial_\tau\psi_a - t_x\left(\bar\psi_a(x+1)\psi_a(x) + \text{c.c.}\right)\right], \\ S_I &= \int d\tau \sum_{x,z}\left[\frac{\Delta^2}{2U} - \left(\bar\psi_a\Delta_{ab}\psi_b - \frac{1}{2}\Delta_{ab}\delta_{ab}\right)\right], \\ S_\perp &= \int d\tau \sum_{x,z} it_\perp(-1)^x\left[\bar\psi_a(z+1)\psi_a(z) + \text{c.c.}\right], \end{aligned} \tag{4}$$

where $\Delta(x,\tau,z)$ is a matrix Hubbard-Stratonovich field. As usual, at a mean-field level $\Delta_{ab}^0 = U\langle\bar\psi_b\psi_a - \delta_{ab}/2\rangle$, which is easily solved to give $\Delta_{ab}^0(x) = \Delta_0(-1)^x\Lambda_{ab}$ with $\Lambda_{ab} = S_p\delta_{ab}$. The massless fluctuations beyond the mean-field theory can be represented by slowly-varying unitary rotations of the "staggered magnetization" Δ_{ab}^0. Ignoring the massive modes associated with the amplitude fluctuations, we write

$$\Delta_{ab}(x,\tau,z) = u_{ac}(x,\tau,z)\Delta_{cd}^0(x)u_{db}^\dagger(x,\tau,z), \quad u \in SU(2n).$$

In terms of the left (ψ_L) and right (ψ_R) moving fermion fields defined in the continuum limit around the Fermi points in the strongly coupled x-direction, the action in Eq. (4) can be written as

$$S_0 = \mathrm{Tr}(\bar\psi_R \partial_- \psi_R + \bar\psi_L \partial_+ \psi_L) - 2i\delta t \mathrm{Tr}(\bar\psi_R \psi_L - \bar\psi_L \psi_R),$$
$$S_I = \mathrm{Tr}(\Delta_0^2/2U) - \Delta_0 \mathrm{Tr}(\bar\psi_R u \Lambda u^\dagger \psi_L + \bar\psi_L u \Lambda u^\dagger \psi_R),$$
$$S_\perp = i t_\perp \mathrm{Tr}\left[\bar\psi_R(z)\psi_L(z+1) + \bar\psi_L(z)\psi_R(z+1) - \mathrm{c.c.}\right].$$

Here Tr stands for the trace over space-time as well as the replica and energy indices, $\partial_\pm = \partial_\tau \pm i v_F \partial_x$ with v_F the Fermi velocity. Next, we perform a local gauge transformation, $\psi'_{L,R} = u^\dagger \psi_{L,R}$, and define the pure SU(2n) gauge fields $A_\pm \equiv -iu^\dagger \partial_\pm u = A_\tau \pm i v_F A_x$. The action becomes (dropping the primes),

$$\begin{aligned} S = {} & \mathrm{Tr}\left[\bar\psi_R(\partial_- + iA_-)\psi_R + \bar\psi_L(\partial_+ + iA_+)\psi_L\right] \\ & + i\Delta' \mathrm{Tr}\left[\bar\psi_R e^{i\Lambda(\pi/2+2\Delta\theta)}\psi_L - \bar\psi_L e^{-i\Lambda(\pi/2+2\Delta\theta)}\psi_R\right] \\ & + \mathrm{Tr}(\Delta_0^2/2U) + S_\perp(\psi_{L,R} \to u^\dagger \psi_{L,R}). \end{aligned} \tag{5}$$

where $\Delta' = (\Delta_0^2 + 4\delta^2 t^2)^{1/2}$ and $2\Delta\theta = \tan^{-1}(2\delta t/\Delta_0)$.

The final step is to integrate out the fermion fields. In order to do so, we need to bring the term proportional to Δ' in Eq. (5) to the standard mass term for Dirac fermions $i\Delta' \mathrm{Tr}\left[\bar\psi_R \psi_L - \bar\psi_L \psi_R\right]$. This can be done by the following chiral gauge transformation,

$$\psi_{R,(L),a} \to e^{(-)i\Lambda_{aa}(\pi/4+\Delta\theta)}\psi_{R,(L),a}. \tag{6}$$

As a result, we encounter the well-known chiral anomaly [12], which arises from the Jacobian associated with the transformation (6) and leads to,

$$S_{\mathrm{chiral}}(A) = \frac{i}{\pi}\left(\frac{\pi}{4} + \Delta\theta\right) \mathrm{Tr}\epsilon_{\mu\nu}\Lambda \partial_\mu A_\nu, \tag{7}$$

where $\mu, \nu = x, \tau$, in the transformed action. Now it is straightforward to integrate out the massive fermions and obtain the effective action in terms of the gauge field. Using the equalities $\mathrm{Tr}\epsilon_{\mu\nu}\Lambda\partial_\mu A_\nu = (i/4)\mathrm{Tr}\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q$ and $\mathrm{Tr}[A_\mu, \Lambda]^2 = \mathrm{Tr}\partial_\mu Q \partial_\mu Q$, where $Q \equiv u\Lambda u^\dagger \in U(2n)/U(n) \times U(n)$, we obtain,

$$S_{\mathrm{eff}} = \frac{\sigma_{xx}^0}{8}\mathrm{Tr}\partial_\mu Q \partial_\mu Q + \frac{\sigma_{xy}^0}{8}\mathrm{Tr}\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q - \frac{\sigma_{zz}^0}{4\lambda^2}\mathrm{Tr}Q(z+1)Q(z) - h\mathrm{Tr}\Lambda Q. \tag{8}$$

Here we have rescaled the coordinates by $\lambda x \to x$, $\lambda v_F \tau \to \tau$ and $z \to z$. The coupling constants $\sigma^0_{\alpha,\beta}$ have the meaning of conductivities defined on the length scale cutoff λ. For the present network model, $\sigma^0_{xx} = (\Delta_0/\sqrt{\pi}\Delta')^2$, $\sigma^0_{xy} = 1/2 + 2\Delta\theta/\pi$, and $\sigma^0_{zz} = (t_\perp \Delta_0/\sqrt{\pi} v_F \Delta')^2$ for small $t_\perp/v_F$ and δ, and $h = \eta\Delta_0/v_F U \lambda^2$. The coupled-layers in the thermodynamic limit is thus described by the zero-temperature properties of the above (2+1)D quantum NLσM.

For $t_\perp = 0$, $\sigma^0_{zz} = 0$. Eq. (8) reduces to N independent 2D NLσMs discovered by Pruisken and coworkers for the single layer IQHE [13]. In this case, the term that couples to σ^0_{xy} becomes a topological quantity which produces the critical fixed points at $(\sigma_{xx}, \sigma_{xy}) = (\mathrm{const}, i+1/2)$ for the plateau transitions, and the stable fixed points at $(\sigma_{xx}, \sigma_{xy}) = (0, i)$ for the quantum Hall states in units of e^2/h. For the network model, the $i = 0 \to 1$ transition happens at $\delta = 0$ where $\sigma^0_{xy} = 1/2$. For $\sigma^0_{xy} \neq 1/2$, the conductances $(\sigma_{xx}, \sigma_{xy})$ flow to $(0, 1)$ for $\delta > 0$ and $(0, 0)$ for $\delta < 0$. At the transition, the dissipative conductance has a critical value $\sigma^c_{xx} \simeq (0.58 \pm .05)$ [14,15]. Notice however, for finite $\sigma^0_{zz} \ll \sigma^0_{xx}$, the system is highly anisotropic but three-dimensional. The σ^0_{xy}-term, having two derivatives, no longer contains nontrivial topological contributions from slowly-varying field configurations on the scale of the interlayer lattice spacing.

We now present a RG study of Eq. (8). For weak interlayer tunnelings, we follow the dimensional crossover analysis used in the O(3) NLσM description of weakly coupled quantum spin chains [16]. The basic idea is that, since $R \equiv \sigma^0_{zz}/\sigma^0_{xx} \ll 1$, it is possible to consider the renormalization of the coupling constants in Eq. (8) in the 2D sector (x, τ) independently until the renormalized couplings become comparable in all directions at a larger length scale λ'. The 3D isotropic RG is switched on beyond λ'. Since the scaling dimension of the Q-field is zero in the replica limit, this crossover takes place when $R\sigma^0_{xx}/\lambda^2 \approx \sigma^{2d}_{xx}(\lambda')/\lambda'^2$. One then takes the continuum limit in the z-direction by absorbing the cutoff λ'^{-2} into defining the derivatives and obtains an isotropic 3D NLσM action (plus the symmetry breaking term),

$$S'_{\mathrm{eff}} = \frac{\sigma_{xx}(\lambda')}{8} \mathrm{Tr} \partial_\rho Q \partial_\rho Q + \frac{\sigma_{xy}(\lambda')}{8} \mathrm{Tr} \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q, \tag{9}$$

with $\rho = x, \tau, z$ and $\sigma_{\alpha\beta}(\lambda') = \sigma^{2d}_{\alpha\beta}(\lambda')/\lambda'$, the conductivities at cutoff λ'. The important

point is that the latter are the bare coupling constants for the subsequent 3D RG [16]. Whether the system is in the insulating or metallic phase is thus determined by the renormalized conductances at the end of the 2D RG.

Since the σ_{xy}-term in the continuum action Eq. (9) is no longer topological, we performed perturbative RG calculations to two-loop order to determine the flow of the conductance parameters. This approach is valid in the metallic phase where the bare conductivity is large for a large number of layers. For general n, we found the recursion relations for the conductivities under RG scale transformation, $\lambda' \to b\lambda'$,

$$\sigma'_{xx} = \sigma_{xx}\left[1 - 2n\frac{1}{\sigma_{xx}}I_d + 2(n^2+1)\frac{\epsilon}{d}\frac{1}{\sigma_{xx}^2}I_d^2\right], \tag{10}$$

$$\sigma'_{xy} = \sigma_{xy}\left[1 - 2n\frac{\epsilon}{d}\frac{1}{\sigma_{xx}}I_d - 8n^2\frac{\epsilon}{d^2}\frac{1}{\sigma_{xx}^2}I_d^2\right], \tag{11}$$

where $I_d = \int_{1/b\lambda'}^{1/\lambda'} d^dp/(2\pi)^d[1/(p^2+h)]$, $\epsilon = d-2$. Notice that the corrections to σ'_{xy} vanish in the replica limit $n \to 0$, *i.e. the Hall conductivity is unrenormalized in the 3D regime*. We will show later that this property is crucial for the quantization of the Hall conductance. This result should be contrasted to the one obtained in weak magnetic fields where the Hall conductivity is found to renormalize in the same way as the dissipative conductivity [17]. Defining the dimensionless conductances $g_{\mu\nu} = \sigma'_{\mu\nu}\lambda'^{\epsilon}b^{\epsilon}$ and $b = e^l$, Eq. (10) leads to the RG equation in the limit $n \to 0$, $dg_{xx}/dl = \epsilon g_{xx} - (4/dK_d^2)g_{xx}^{-1}$, $K_d = 2^{d-1}\pi^{d/2}\Gamma(d/2)$, consistent with the known result for the unitary NLσM without the σ_{xy}-term in $2+\epsilon$ expansions [18]. For $d=3$, there is a nontrivial critical fixed point at $g_c = \sqrt{4/dK_d^2\epsilon} = 1/\sqrt{3}\pi^2$. It separates a metallic phase with $g_{xx}(\lambda')/g_c > 1$ from an insulating phase, where $g_{xx}(\lambda')/g_c < 1$. The dissipative conductivity vanishes at the metal-insulator transition whereas the Hall conductivity remains close to its bare value at the beginning of the 3D RG. The Hall conductance follows the simple Ohm's law, $dg_{xy}/dl = (d-2)g_{xy}$.

We now discuss the phase structure assuming the carrier density in each layer to be nominally the same. (i) For $\delta = 0$, $\sigma_{xy}^0 = 1/2$. The individual layers are at the critical point for the 2D plateau transition. During the initial 2D RG, σ_{xy}^{2d} does not renormalize and σ_{xx}^{2d} flows towards its finite critical value which is of order one. Thus the crossover

length $\lambda' \approx \lambda/\sqrt{R}$. For small interlayer tunneling $R \ll 1$, $\lambda' \gg \lambda$ such that $\sigma_{xx}^{2d}(\lambda')$ flows towards the fixed point value $\sigma_{xx}^{2d}(\infty) = \sigma_{xx}^{c} \simeq 0.55$ [14,15]. The latter is greater than the 3D critical conductance g_c derived above. Thus we conclude that when the Fermi energy is located at the critical point for the 2D plateau transition, an arbitrarily small interlayer tunneling leads to a 3D metallic state. A unique feature of the metallic phase is that the Hall conductivity is unrenormalized and remains close to $\sigma_{xy}(\lambda')$ down to low temperatures. (ii) For $\delta \neq 0$, the 2D RG scales towards the insulator/quantum Hall fixed points, *i.e.* $\sigma_{xx}^{2d} \to 0$, whereas $\sigma_{xy}^{2d} \to 0$ ($\delta < 0$) and 1 ($\delta > 0$) around the first plateau transition. The metallic phase is stable so long as $\sigma_{xx}^{2d}(\lambda') > g_c$. Clearly, with decreasing (increasing) $t_\perp$ ($|\delta|$), at a critical $t_\perp^c$ (δ_c) where $\sigma_{xx}^{2d}(\lambda') = g_c$, a metal-insulator transition takes place. For $t_\perp < {t_\perp}^c$ (or $|\delta| > \delta_c$), the system is in the 3D insulating phase. To determine the phase boundary, notice that for $\delta \neq 0$, a finite localization length develops in the 2D sector, $\xi_{2d} \propto |\delta|^{-\nu_{2d}}$ with $\nu_{2d} \simeq 7/3$ [7,10,15]. Thus the 2D RG flow stops at ξ_{2d} beyond which a gap would develop in the 2D sector. Setting the crossover length $\lambda' = \xi_{2d}$, one finds that the critical anisotropy $R_c \sim (\lambda/\xi_{2d})^2$, leading to the phase boundary $t_\perp^c \propto t|\delta|^{\nu_{2d}}$. The width of the metallic phase is then given by $W_\delta \propto (t_\perp/t)^{1/\nu_{2d}}$, consistent with the numerical results of Chalker and Dohmen [8]. (iii) From the above discussion, it is clear that the metal-insulator transition is in the 3D unitary universality class of the Anderson transition since the Hall conductivity appears to be a 3D RG invariant. The two-loop RG equations imply that the 3D localization length diverges as $\xi_{3d} \propto |g_{xx} - g_c|^{-\nu_{3d}}$, $\nu_{3d} = 1/2\epsilon$. Simulations of various unitary models give $\nu_{3d} = 1.35 \pm .15$ [19] which indeed agrees with the numerical value $1.45 \pm .25$ obtained directly from the layered network model [8].

We next discuss the quantization of the Hall conductance in the insulating phases. In this case, during the first stage of the RG, the Hall conductance σ_{xy}^{2d} in Eq. (9) flows towards the 2D quantized values, *i.e.* $\sigma_{xy}^{2d}(\lambda') \to ie^2/h$, for large anisotropy ($R \ll 1$) such that $\lambda' \to \infty$. The quantization of the 3D Hall conductance is then possible provided that σ_{xy} in Eq. (9) does not renormalize in the 3D regime. Restoring the discrete sum in the z-direction, *i.e.*,

$(1/\lambda') \int dz \to \sum_z$, this term becomes $S_{xy} = N\sigma_{xy}^{2d}(\lambda') \int dx d\tau \text{tr} \epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q$, leading to the Hall conductance quantization $\sigma_{xy} \to iNe^2/h$. The 3D quantum Hall states are therefore characterized by the resistance behaviors $\rho_{xx}, \rho_{yy} \to 0$, $\rho_{zz} \to \infty$, and $\rho_{xy} = (iN)^{-1}h/e^2$. Due to the intervening metallic phases, the transitions between the quantized Hall plateaus comprise an insulator-metal and a subsequent metal-insulator transition, and have a finite width at low-temperatures. These results are consistent with the experimental observations of the IQHE in the 30-layer [3] and the more recent 200-layer GaAs/AlGaAs structures [4].

In contrast to 2D, where the electronic states are localized at all energies except a critical set of zero measure, it is the existence of metal-insulator transitions and the absence of localization corrections to the Hall conductivity that give rise to the quantization of the Hall conductance in weakly coupled layered systems. This is supported by our two-loop RG results in Eqs (10) and (11). Although these results do not form a proof to all orders in the perturbative RG, we believe that the evidence is sufficiently strong.

We now briefly discuss the possible topological effects not included in the present analysis. The discrete (in the z-direction) action in Eq. (8) allows contributions from topologically-stable, point-singular field configurations, *i.e.* the hedgehogs at which the instanton number changes abruptly from one layer to the next [20]. These contributions could in principle enter during the first stage of the RG and modify the bare parameters of the continuum action in Eq. (9). The precise effects of the hedgehogs in the replica limit is not understood at the present time. Nevertheless we do not expect them to change the main results discussed above.

Finally, we consider the surface states in the quantum Hall phases where the bulk localization length is very short. The edge state supported by each layer couples together and forms an interesting 2D surface system decoupled from the bulk [8,21]. In the presence of boundaries, in addition to the bulk σ_{xy}-term, the action in Eq. (7) leads to an additional contribution $(i\sigma_{xy}/2) \oint dr_\mu \text{tr} \Lambda A_\mu$, where the integral is over the boundary of the sample at $(x = 0, L)$ while keeping the periodic boundary condition in τ. It is easy to show that this surface term can be written in terms of $Q(u, \tau, z)$, a smooth homotopy between

$Q(u=0)=Q(x=0)$ and $Q(u=1)=Q(x=L)$. The action on the surface is then,

$$S_{\rm sf}=\frac{\sigma_{xy}}{4}\int_0^1 du{\rm Tr}Q\partial_u Q\partial_\tau Q+\frac{\sigma_{zz}^0}{8}{\rm Tr}\partial_z Q\partial_z Q. \tag{12}$$

This action is identical to the coherent state path integral action of an SU(2n) ferromagnetic Heisenberg spin chain with spin $S=\sigma_{xy}/2$ and exchange $J=-\sigma_{zz}^0/\sigma_{xy}^2$. The first term in Eq. (12) corresponds to the Berry phase term. The spin quantization in this case results from the Hall conductance quantization in the quantum Hall state. The equations of motion of $S_{\rm sf}$ give the exactly known one-magnon dispersion valid for all n, *i.e.* $iq_\tau=|J|Sq_z^2$. By the analogy between the spin-spin correlation function and the edge electron two-particle Green's function [10], it can be shown that this mode corresponds to the anisotropic diffusive mode on the surface, $i\omega\rho=-iq_\tau+(\sigma_{zz}^0/2\sigma_{xy})q_z^2$. Following Wegner [22], the latter leads to the conductivities on the surface: $\sigma_{zz}^{\rm sf}=\sigma_{zz}^0/2\sigma_{xy}$ and $\sigma_{\tau\tau}^{\rm sf}(\omega)\propto i/\omega$. The present approach to the chiral surface state is complimentary to those formulated using supersymmetric fields [23].

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